

SEQUENCES AND SERIES

- 1 For each of the following geometric series, write down the common ratio and find the value of the eighth term.
- a** $3 + 9 + 27 + 81 + \dots$ **b** $1024 + 256 + 64 + 16 + \dots$ **c** $1 - 2 + 4 - 8 + \dots$
- 2 For each of the following geometric series, find an expression for the n th term.
- a** $1 + 5 + 25 + 125 + \dots$ **b** $3 - 12 + 48 - 192 + \dots$ **c** $81 + 54 + 36 + 24 + \dots$
- 3 Find the sum of the first 12 terms of each of the following geometric series.
- a** $2 + 4 + 8 + 16 + \dots$ **b** $640 + 320 + 160 + 80 + \dots$ **c** $\frac{1}{6} - \frac{1}{2} + 1\frac{1}{2} - 4\frac{1}{2} + \dots$
- 4 Given the first term, a , the common ratio, r , and the number of terms, n , find the sum of each of the following geometric series. Give your answers to 3 decimal places where appropriate.
- a** $a = 4, r = 3, n = 8$ **b** $a = 48, r = \frac{1}{2}, n = 14$ **c** $a = -1, r = -4, n = 12$
- d** $a = 200, r = 0.7, n = 20$ **e** $a = 120, r = -\frac{3}{4}, n = 15$ **f** $a = -25, r = 1.2, n = 30$
- 5 Evaluate to an appropriate degree of accuracy
- a** $\sum_{r=1}^9 3^r$ **b** $\sum_{r=1}^6 8^{r+1}$ **c** $\sum_{r=1}^{10} (10 \times 2^r)$ **d** $\sum_{r=1}^8 (0.8)^r$
- e** $\sum_{r=1}^{10} \left[12 \times \left(\frac{1}{6}\right)^r \right]$ **f** $\sum_{r=1}^9 (-4)^r$ **g** $\sum_{r=4}^{20} \left(\frac{1}{2}\right)^r$ **h** $\sum_{r=3}^9 \left[2 \times (-3)^r \right]$
- 6 The second and third terms of a geometric series are 2 and 10 respectively.
- a** Find the common ratio of the series.
b Find the first term of the series.
c Find the sum of the first eight terms of the series.
- 7 The first and fourth terms of a geometric series are 2 and 54 respectively.
- a** Find the common ratio of the series.
b Find the ninth term of the series.
- 8 The third and fourth terms of a geometric series are 24 and 8 respectively.
- a** Find the common ratio of the series.
b Find the first term of the series.
c Find, to 3 decimal places, the sum of the first 11 terms of the series.
- 9 The first and third terms of a geometric series are 6 and 24 respectively.
- a** Find the two possible values for the common ratio of the series.
 Given also that the common ratio of the series is positive,
b find the sum of the first 15 terms of the series.
- 10 The first and fourth terms of a geometric series are 768 and -96 respectively.
- a** Find the common ratio of the series.
b Find the tenth term of the series.

SEQUENCES AND SERIES

continued

- 11** The second and fifth terms of a geometric series are 0.5 and 32 respectively.
- Find the first term and common ratio of the series.
 - Find the number of terms of the series that are smaller than 10 000.
- 12** The sum of the first four terms of a geometric series is 130 and its common ratio is $1\frac{1}{2}$.
- Find the first term of the series.
 - Find the eighth term of the series.
 - Find the least value of n for which the sum of the first n terms of the series is greater than 30 000.
- 13** All the terms of a geometric series are positive. The sum of the first and second terms of the series is 10.8 and the sum of the third and fourth terms of the series is 43.2
- Find the first term and common ratio of the series.
 - Find the sum of the first 16 terms of the series.
- 14** For each of the following geometric series, either find its sum to infinity or explain why this cannot be found.
- $12 + 6 + 3 + 1.5 + \dots$
 - $270 + 90 + 30 + 10 + \dots$
 - $25 - 30 + 36 - 43.2 + \dots$
 - $216 + 144 + 96 + 64 + \dots$
 - $\frac{8}{25} + \frac{2}{5} + \frac{1}{2} + \frac{5}{8} + \dots$
 - $500 - 300 + 180 - 108 + \dots$
- 15** Find the sum to infinity of the geometric series with n th term
- $(0.9)^n$
 - $6 \times (\frac{1}{2})^n$
 - $(-\frac{3}{4})^{n-1}$
 - $40 \times (0.8)^n$
- 16** A geometric series has first term 80 and common ratio 0.2
- Find the sum to infinity of the series.
 - Find the difference between the sum to infinity of the series and the sum of the first six terms of the series.
- 17** A sequence is defined by the recurrence relation
- $$u_{n+1} = \frac{1}{3}u_n, \quad n > 0, \quad u_1 = 1.$$
- Write down the first four terms of the sequence.
 - Evaluate $\sum_{r=1}^{\infty} u_r$.
- 18** The common ratio of a geometric series is 0.55 and the sum to infinity of the series is 40.
- Find the first term of the series.
 - Find the smallest value of n for which the n th term of the series is less than 0.001
- 19** The sum, S_n , of the first n terms of a geometric series is given by $S_n = 2^n - 1$.
- Find the first term and the fifth term of the series.
 - Find an expression for the n th term of the series.
- 20** The first three terms of a geometric series are $(k + 10)$, k and $(k - 6)$ respectively.
- Find the value of the constant k .
 - Find the sum to infinity of the series.